Yaroslavl State University
Preprint YARU-HE-95/01

hep-ph/9503239

The Minimal Quark-Lepton Symmetry Model and the Limit on Z'-mass.

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Abstract

A minimal extension of the Standard Model containing the four-color quark-lepton symmetry is proposed and discussed. The existence of a rather light extra Z'-boson originated from the four-color quark-lepton symmetry is shown to be compatible with the current electroweak data. The cross sections $\sigma(e^+e^- \to \gamma, Z, Z' \to \bar{f}f)$ are calculated, and their deviations from the SM predictions are shown to be significant at $\sqrt{s} \geq 200\,GeV$ and available for the measurements at the LEP200 and future colliders.

To be published in Phys. Lett. B

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The successful unification of the electromagnetic and weak interactions by the Standard Model (SM) and the idea of the possible existence of the more large symmetries at high energies stimulate the search for a new physics at the present and future colliders. It seems that high symmetries will manifest themselves consecutively one after an other as the energies of colliders increase. The $SU(2) \times U(1)$-symmetry of the SM is, probably, only the first such symmetry, what is the next one then? To answer this question it seems reasonable to investigate various minimal extensions of the SM by adding to it some additional symmetries such as the right $SU_R(2)$-symmetry in the $SU_L(2) \times SU_R(2) \times U(1)$-models, the supersymmetry in the Minimal Supersymmetric Standard Model (MSSM), etc. One of such symmetries possibly existing in nature and being worthy of the detailed investigation now is the four-color quark-lepton symmetry regarding the lepton number as the fourth color [1].

In this work we propose the minimal quark-lepton symmetry model of the unification of the strong and electroweak interactions (MQLS-model) which is the minimal extension of the SM containing the four-color quark-lepton symmetry. We discuss some features of the extra Z'-bosons originated from this symmetry.

The model to be discussed here is based on the $SU_V(4) \times SU_L(2) \times U_R(1)$-group as the minimal group containing the four-color symmetry of quarks and leptons. In this model the quarks $Q_{pa\alpha} = \psi_{pa\alpha}$, $a = 1, 2$, $\alpha = 1, 2, 3$ and the corresponding leptons $\ell_{pa} = \psi_{pa4}$ in each generation of the number $p = 1, 2, 3, \ldots$ form the four-color fundamental quartet ψ_{paA}, $A = 1, 2, 3, 4$ of the $SU_V(4)$-group. Under the $SU_L(2) \times U_R(1)$-group the left fermions are the doublets with $Y_L = 0$ and the right fermions are the singlets with $Y_R = \pm 1$ for the "up" ($a = 1$) and "down" ($a = 2$) fermions respectively. For three generations the basic up- and down- fermion $SU_V(4)$-quartets are

$$\psi'_{p1A} \; : \quad \begin{pmatrix} u'_\alpha \\ \nu'_e \end{pmatrix}, \begin{pmatrix} c'_\alpha \\ \nu'_\mu \end{pmatrix}, \begin{pmatrix} t'_\alpha \\ \nu'_\tau \end{pmatrix}, \; \ldots$$

$$\psi'_{p2A} \; : \quad \begin{pmatrix} d'_\alpha \\ e^{-\prime} \end{pmatrix}, \begin{pmatrix} s'_\alpha \\ \mu^{-\prime} \end{pmatrix}, \begin{pmatrix} b'_\alpha \\ \tau^{-\prime} \end{pmatrix}, \; \ldots$$

where the basic quark and lepton fields ${Q'}^{L,R}_{pa\alpha}$, ${\ell'}^{L,R}_{pa}$ can be written, in general, as superpositions

$$Q'^{L,R}_{pa\alpha} = \sum_q \left(A^{L,R}_{Q_a}\right)_{pq} Q^{L,R}_{qa\alpha}, \qquad \ell'^{L,R}_{pa} = \sum_q \left(A^{L,R}_{\ell_a}\right)_{pq} \ell^{L,R}_{qa}$$

of mass eigenstates $Q^{L,R}_{qa\alpha}$, $\ell^{L,R}_{qa}$. Here $A^{L,R}_{Q_a}$ and $A^{L,R}_{\ell_a}$ are unitary matrices diagonalizing the mass matrices of quarks and leptons respectively. The electric charges of quarks and leptons are related to the generators of the group by

$$Q^{em}_{L,R} = \sqrt{\frac{2}{3}}\, t^{L,R}_{15} + \frac{\tau^L_3}{2} + \frac{Y^R}{2}\,,$$

where t_{15}, $\tau_3/2$ are the corresponding generators, τ_3 is the Pauli matrix.

According to the structure of the group the gauge sector consists of 19 fields A^i_μ, $i = 1, 2, \ldots, 15$, W^k_μ, $k = 1, 2, 3$ and B_μ. The first eight of them are the gluons $G^j_\mu = A^j_\mu$, $j = 1, 2, \ldots, 8$, the next six fields form the triplets of the leptoquarks $V^\pm_{\alpha\mu}$, $\alpha = 1, 2, 3$ with the electric charges $Q^{em}_V = \pm 2/3$, W^1_μ, W^2_μ form the $W^\pm$-bosons in a usual way and the remained fields A^{15}_μ, W^3_μ, B_μ form the photon, the Z-boson and an extra Z'-boson.

The electromagnetic field A_μ is related to A^{15}_μ, W^3_μ, B_μ by

$$A_\mu = s_S A^{15}_\mu + \sqrt{1 - s^2_W - s^2_S}\, B_\mu + s_W W^3_\mu,$$

and two orthogonal to A_μ fields $Z_{1\mu}$ and $Z_{2\mu}$ can be written as

$$\begin{aligned} Z_{1\mu} &= -t_W(s_S A^{15}_\mu + \sqrt{1 - s^2_W - s^2_S}\, B_\mu) + c_W W^3_\mu, \\ Z_{2\mu} &= (\sqrt{1 - s^2_W - s^2_S}\, A^{15}_\mu - s_S B_\mu)/c_W, \end{aligned}$$

where $s_{W,S} = \sin\theta_{W,S}$, $c_W = \cos\theta_W$, $t_W = \tan\theta_W$. The angles θ_W and θ_S of the weak and strong mixings are defined as

$$s^2_W = \frac{\alpha(m)}{\alpha_W(m)}, \tag{1}$$

$$s^2_S = \frac{2}{3}\frac{\alpha(m)}{\alpha_{15}(m)} = \frac{2}{3}\frac{\alpha(m)}{\alpha_S(m)}\left[1 + \frac{\alpha_S(m)}{2\pi}\left(b \ln\frac{M_C}{m} + b_{15} \ln\frac{M'}{m}\right)\right], \tag{2}$$

where $\alpha(m)$, $\alpha_W(m)$, $\alpha_S(m)$ are the electromagnetic, weak and strong coupling constants at the scale m, M_C is the mass scale of the $SU_V(4)$-symmetry breaking, M' is the possible intermediate mass scale of $U_{15}(1)$-symmetry breaking, $b = b_S - b_{15}$, b_S and b_{15} are the group constants for the group $SU_c(3)$ and $U_{15}(1)$ respectively. Taking into account the gauge and fermion fields gives $b_S = 11 - (4/3)n_G$, $b_{15} = -(4/3)n_G$ and $b = 11$, n_G is the number of fermion generations with masses below M_C. The last equality in (2) is obtained by the elimination of the $SU_V(4)$ unified gauge coupling constant $\alpha_4(M_C) = g_4^2/4\pi$ from the one-loop approximation relations

$$\alpha_{S,15}(m) \;=\; \alpha_4(M_C)/(1 - \frac{\alpha_4(M_C)}{2\pi}\, b_{S,15}\, \ln\frac{M_C}{m}) \tag{3}$$

between $\alpha_4(M_C)$ and the A^{15}-interaction constant $\alpha_{15}(m)$ and $\alpha_S(m)$.

The interaction of the gauge fields with the fermions has the form

$$\begin{aligned}
\mathcal{L}_{\psi}^{gauge} \;=\;& \frac{g_4}{\sqrt{2}}\{V_\mu^\alpha[\bar{Q}^L_{pa\alpha}\gamma^\mu (K_a^L)_{pq}\ell^L_{qa} + \bar{Q}^R_{pa\alpha}\gamma^\mu (K_a^R)_{pq}\ell^R_{qa}] + h.c.\} \\
+\;& \frac{g_2}{\sqrt{2}}\{W_\mu^+[\bar{Q}^L_{p1\alpha}\gamma^\mu (C_Q)_{pq} Q^L_{q2\alpha} + \bar{\ell}^L_{p1}\gamma^\mu (C_\ell)_{pq}\ell^L_{q2}] + h.c.\} \\
+\;& g_{st} G_\mu^j(\bar{Q}\gamma^\mu t_j Q) - |e| A_\mu(\bar{\psi}\gamma^\mu Q^{em}\psi) + \mathcal{L}_{NC}^{gauge}.
\end{aligned} \tag{4}$$

Here the first term describes the interaction of leptoquarks with quarks and leptons by the constant g_4 related to $\alpha_S(m)$ by (3). This interaction contains, in general, the new generation mixing due to the matrices $K_a^{L,R} = (A^{L,R}_{Q_a})^+ A^{L,R}_{\ell_a}$. These matrices should be extracted from the experiments with the leptoquarks. Some restrictions on these matrices and on the leptoquark masses resulting from the rare K, π and B decays in the case of $K_a^L = K_a^R$ have been investigated recently in Ref. [2, 3]. The second term in (4) describes the weak charged current interaction of $W^\pm$-bosons with quarks or leptons by the constant g_2 related to the Fermi constant G_F and m_W in a usual way. This interaction contains the well known Cabibbo-Kobayashi-Maskawa mixing of the quarks due to the CKM matrix $C_Q = (A^L_{Q_1})^+ A^L_{Q_2}$ and, in general, the analogous mixing in the lepton sector due to the lepton mixing matrix $C_\ell = (A^L_{\ell_1})^+ A^L_{\ell_2}$. The next two terms are the QCD- and QED- interactions. The neutral current interaction with the gauge fields $\mathcal{L}_{NC}^{gauge}$ can be written as

$$\mathcal{L}_{NC}^{gauge} = -Z_\mu J_\mu^Z - Z'_\mu J_\mu^{Z'}, \tag{5}$$

where

$$\begin{aligned} Z_\mu &= Z_{1\mu}\cos\theta_m + Z_{2\mu}\sin\theta_m, \\ Z'_\mu &= -Z_{1\mu}\sin\theta_m + Z_{2\mu}\cos\theta_m \end{aligned}$$

are the mass eigenstate fields and

$$J_\mu^Z = J_\mu^{Z_1}\cos\theta_m + J_\mu^{Z_2}\sin\theta_m, \tag{6}$$

$$J_\mu^{Z'} = -J_\mu^{Z_1}\sin\theta_m + J_\mu^{Z_2}\cos\theta_m, \tag{7}$$

$$J_\mu^{Z_1} = \frac{|e|}{s_W c_W}\,(J_\mu^{3L} - s_W^2 J_\mu^{em}), \tag{8}$$

$$J_\mu^{Z_2} = \frac{|e|}{s_S c_W\sqrt{1 - s_W^2 - s_S^2}}\left[c_W^2\sqrt{\frac{2}{3}}J_\mu^{15} - s_S^2(J_\mu^{em} - J_\mu^{3L})\right] \tag{9}$$

with the currents $J_\mu^{em} = (\bar\psi_{paA}\gamma_\mu Q_{aA}^{em}\psi_{paA})$, $J_\mu^{3L} = \frac{1}{2}(\bar\psi_{paA}\gamma_\mu(1+\gamma_5)(\tau_3/2)_{aa}\psi_{paA})$, $J_\mu^{15} = (\bar\psi_{paA}\gamma_\mu(t_{15})_{AA}\psi_{paA})$. The Z_1-current (8) is the usual neutral current of the Standard Model, but the structure of the Z_2-current (9) is specified by the model under consideration. The $Z - Z'$-mixing angle θ_m is defined by the symmetry breaking mechanism of the model and is found to be small.

The Higgs sector of the model is taken in the simplest way and consists of the four multiplets (4, 1, 1), (1, 2, 1), (15, 2, 1), (15, 1, 0) of $SU_V(4) \times SU_L(2) \times U_R(1)$-group with the vacuum expectation values (VEV's) $\langle\phi_A^{(1)}\rangle = \delta_{A4}\eta_1/\sqrt{2}$, $\langle\phi_a^{(2)}\rangle = \delta_{a2}\eta_2/\sqrt{2}$, $\langle\phi_{ia}^{(3)}\rangle = \delta_{i15}\delta_{a2}\eta_3$, $i = 1, 2, \ldots, 15$ and $\langle\phi_i^{(4)}\rangle = \delta_{i15}\eta_4$ respectively. The field $\phi^{(1)}$ breaks the $SU_V(4)$- symmetry down to $SU_C(3)$ giving the masses to the Z'- boson and to the leptoquarks. The $SU_L(2)$- doublet $\phi^{(2)}$ breaks the $SU_L(2)$- symmetry in the usual way and gives the equal masses to the fermions belonging to the same generation. The main function of the $\phi^{(3)}$ multiplet is to split the masses of the quarks and leptons in each generation. The $\phi^{(4)}$ multiplet breaks the $SU_V(4)$- symmetry down to $SU_C(3) \times U_{15}(1)$ contributing to the leptoquark masses and splitting them from the Z'- mass. After breaking the symmetry in such a way the masses

of quarks and leptons are defined by VEV's η_2, η_3 and by Yukawa coupling constants and can be arbitrary just as they are in the Standard Model, the photon and the gluons are still massless but all the other gauge fields acquire the masses.

The model has, in general, three mass scales determined by the VEV's $\eta = \sqrt{\eta_2^2 + \eta_3^2}$, η_1 and η_4 respectively. The first mass scale is the usual mass scale of the Standard Model $\eta = (\sqrt{2}G_F)^{-1/2} \simeq 250\, GeV$, the second one is the possible intermediate mass scale $M' \sim m_{Z'}$ of $U_{15}(1)$-symmetry breaking and the third one is the mass scale $M_C \sim m_V$ of $SU_V(4)$- symmetry breaking. The low limit on M_C can be about a few hundreds TeV or, possibly, somewhat lower [2, 3] in dependence on the character of the $K_a^{L,R}$ mixing in the leptoquark interaction in (4).

In the case of $\eta_4 \gg \eta_1$ the symmetry breaking has the three stage form

$$
\begin{aligned}
SU_V(4) \times SU_L(2) \times U_R(1) &\xrightarrow{\eta_4} SU_C(3) \times U_{15}(1) \times SU_L(2) \times U_R(1) \\
&\xrightarrow{\eta_1} SU_C(3) \times SU_L(2) \times U(1) \xrightarrow{\eta} SU_C(3) \times U_{em}(1).
\end{aligned}
$$

Because the VEV η_4 contributes only to the masses of the leptoquarks the Z'-boson can be rather light ($m_{Z'} \sim M' \sim \eta_1$) in this case with the leptoquarks being sufficiently heavy ($m_V \sim M_C \sim \eta_4$).

In another limiting case of $\eta_4 = 0$ the model has only two mass scales η and $M' \sim M_C \sim \eta_1$, the symmetry breaking has two stages

$$
SU_V(4) \times SU_L(2) \times U_R(1) \xrightarrow{\eta_1} SU_C(3) \times SU_L(2) \times U(1) \xrightarrow{\eta} SU_C(3) \times U_{em}(1)
$$

and Z'-boson must be as heavy as the leptoquarks are: $m_{Z'} \sim m_V \sim M_C \sim M' \sim \eta_1$.

Irrespective of the hierarchy of VEV's η_1 and η_4 the model predicts the relation between the masses of the W-, Z- and Z'- bosons

$$
(\mu^2 - \rho_0)(\rho_0 - 1) = \rho_0^2 \sigma^2, \tag{10}
$$

where $\mu \equiv m_{Z'}/m_Z$, $\rho_0 \equiv m_W^2/m_Z^2 c_W^2$ and

$$
\sigma = \frac{s_W s_S}{\sqrt{1 - s_W^2 - s_S^2}}. \tag{11}
$$

Simultaneously the model gives for the Z - Z' mixing angle θ_m the expression

$$\sin\theta_m = \Big[1 + \Big(\frac{\rho_0\sigma}{\rho_0 - 1}\Big)^2\Big]^{-1/2} \tag{12}$$

For $\theta_m \ll 1$ and $\rho_0 \simeq 1$ we also obtain from (10), (12) that $\theta_m \simeq \sigma\, m_Z^2/m_{Z'}^2$.

It should be noted for comparision that the extended gauge model based on the $SU_L(2)\times U(1)\times U'(1)$- group contains the Z'- boson mass and the Z - Z' mixing angle as the independent and arbitrary parameters because the corresponding to the additional group $U'(1)$ coupling constant is arbitrary in this model. Unlike this the $U_{15}(1)$ coupling constant g_{15} of the MQLS model is related to the strong coupling constant α_S by (3), which leads as a result to the relations (10), (11).

The Yukawa interaction of the fermions with the scalar fields contains only the $SU_L(2)$- doublets $\phi^{(2)}$ and $\phi_i^{(3)}$ and has, in general, the form

$$\mathcal{L}_\psi^{Yukawa} = -\bar{\psi}'^L_{paA}[(h_b)_{pq}\phi_a^{(2)b}\delta_{AB} + (h'_b)_{pq}\phi_{ia}^{(3)b}(t_i)_{AB}]\psi'^R_{qbB} + h.c., \tag{13}$$

where $\phi_a^{(2)2} = \phi_a^{(2)}$, $\phi_a^{(2)1} = \varepsilon_{ac}(\phi_c^{(2)})^*$, $\phi_{ia}^{(3)2} = \phi_{ia}^{(3)}$, $\phi_{ia}^{(3)1} = \varepsilon_{ac}(\phi_{ic}^{(3)})^*$, $i = 1, 2, \ldots, 15$, ε_{ac} is antisymmetrical symbol, h_b and h'_b are arbitrary matrices.

After breaking the symmetry the Lagrangian (13) gives the arbitrary masses to the quarks and leptons splitting them in each generation. The remained part of the Lagrangian describes the interaction of the fermions with the scalar fields enterring into $\phi^{(2)}$- and $\phi^{(3)}$- multiplets. Among these fields there are the $SU_L(2)$-down-fields: two triplets of the scalar leptoquarks with the electric charges $Q^{em} = \pm 2/3$, the $SU_C(3)$-octet of the neutral "scalar gluons" and four neutral fields contained in $\phi_2^{(2)}$, $\phi_{15,2}^{(3)}$, and the $SU_L(2)$-up-partners of all these fields. All the scalar fields are massive, some of them can be sufficiently heavy due to the mass scales η_1 and η_4.

The Lagrangaian (13) contains, in particular, the neutral current interaction with the scalar fields. In the unitary gauge one of the four neutral fields can be eliminated and the neutral current interaction of the fermions with the remained three scalar fields χ_1, χ_2 and ω_2 can be written as

$$\begin{aligned} \mathcal{L}_{NC}^{scalar} &= \mathcal{L}_{NC}(\chi_1) + \mathcal{L}_{NC}(\chi_2, \omega_2), \\ \mathcal{L}_{NC}(\chi_1) &= -\frac{\chi_1}{\eta}(\bar{Q}_a M_{Q_a} Q_a + \bar{\ell}_a M_{\ell_a}\ell_a), \end{aligned} \tag{14}$$

$$\begin{aligned}
\mathcal{L}_{NC}(\chi_2,\omega_2) &= \frac{\chi_2 - i\omega_2(\tau_3)_{aa}}{\eta}\frac{1}{2\sin 2\beta} \\
&\times \{\bar{Q}_a^L\left[(1-2\cos 2\beta)M_{Q_a} + K_a^L M_{\ell_a}(K_a^R)^+\right]Q_a^R \\
&+ \bar{\ell}_a^L\left[(-1-2\cos 2\beta)M_{\ell_a} + 3(K_a^L)^+ M_{Q_a}K_a^R\right]\ell_a^R\} + h.c. \quad (15)
\end{aligned}$$

where $(M_{Q_a})_{pq} = m_{Q_{ap}}\delta_{pq}$ and $(M_{\ell_a})_{pq} = m_{\ell_{ap}}\delta_{pq}$ are the diagonal quark and lepton mass matrices, β is the $\phi^{(2)}$ - $\phi_{15}^{(3)}$ mixing angle, $\tan\beta = \eta_3/\eta_2$.

One can see from (14) that χ_1 is the usual Higgs field of the Standard Model. The interaction (15) contains the flavour diagonal interactions represented by the first terms and by the diagonal matrix elements of the second terms in the square brackets. Besides, the interaction (15) contains, in general, also the flavour changing neutral current (FCNC) interactions described by the nondiagonal matrix elements of the second terms in the square brackets. It is interesting that FCNC interaction of the leptons depends on the masses of the quarks and vice versa. It should be noted that the "dangerous" FCNC interactions can be sufficiently supressed by the smallness of the corresponding nondiagonal elements of the $K_a^{L,R}$-matrices and (or) by the large masses of the χ_2 and ω_2 fields. It is pertient to note the particular case of $A_{Q_a}^{L,R} = A_{\ell_a}^{L,R}$ when $K_a^{L,R} = I$, $C_\ell = C_Q$ (without any contradiction with the experiment) and the FCNC interactions in (15) are absent at all.

The doublets $\phi^{(2)}$, $\phi_i^{(3)}$, $i = 1, 2, \ldots, 15$ interact with the photon, $W^\pm$- and Z-bosons and, in general, will contribute into the radiative correction parameters S, T and U of Ref. [4]. The immediate calculation of the contributions $S^{(\phi)}$, $T^{(\phi)}$ and $U^{(\phi)}$ of one scalar doublet ϕ with the standard model hypercharge Y^{SM} into S, T and U parameters yields

$$S^{(\phi)} = -\frac{Y^{SM}}{12\pi}\ln\frac{m_1^2}{m_2^2}, \tag{16}$$

$$T^{(\phi)} = \frac{1}{16\pi s_W^2 c_W^2 m_Z^2}\left[m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2}\ln\frac{m_1^2}{m_2^2}\right], \tag{17}$$

$$\begin{aligned}
U^{(\phi)} &= \frac{1}{12\pi}\left[-\frac{5m_1^4 - 22m_1^2 m_2^2 + 5m_2^4}{3(m_1^2 - m_2^2)^2}\right. \\
&+ \left.\frac{m_1^6 - 3m_1^4 m_2^2 - 3m_1^2 m_2^4 + m_2^6}{(m_1^2 - m_2^2)^3}\ln\frac{m_1^2}{m_2^2}\right], \quad (18)
\end{aligned}$$

where m_1 and m_2 are the masses of the up and down components of the doublet ϕ. It should be noted that the contribution (16) differs essentially from the standard fermion doublet contribution [4]. In particular, it does not contain the mass independent term $(6\pi)^{-1}$ and is not positive defined. The contribution (17) coincides with that from standard fermion doublet, whereas the contribution (18) is less than fermionic one by a factor two.

Applying the formulae (16)–(18) to the multiplets $\phi^{(2)}$ and $\phi^{(3)}$ one should keep in mind that the multiplet $\phi^{(3)}$ contains, in general, eight doublets with $Y^{SM} = 1$, three doublets with $Y^{SM} = 7/3$, three doublets with $Y^{SM} = -1/3$ and the doublet $\phi^{(3)}_{15}$ which together with the doublet $\phi^{(2)}$ forms the standard Higgs doublet and an additional scalar doublet with $Y^{SM} = 1$. The resulting contribution of $\phi^{(2)}$- and $\phi^{(3)}$-multiplets into S, T and U defined by these values of the hypercharge and by the masses of the corresponding up and down fields will satisfy the current constarints [5] on S, T, U if the mass splittings of the scalar doublets are sufficiently small. In particular case of the degenerate scalar doublets their contributions into S, T, U, unlike the standard fermion doublet case, are equal to zero.

The mass relation (10) gives the limit on the Z'-mass. Using the experimental values of G_F, m_W and $\alpha(m_Z)$ [5] we have $s_W^2 = 0.2298 \pm 0.0014$. Then taking the most stringent limit $M_C \geq 10^5 \div 10^6\, GeV$ resulting from $Br(K_L^0 \to \mu e) < 0.94 \cdot 10^{-10}$ [5] into account and using the experimental values $\alpha_s(m_Z) = 0.117 \pm 0.005$ [5] we evaluate s_S^2 and σ from (2) and (11) for $M_C = 10^6 \div 10^{14}\, GeV$ and $M' \ll M_C$ (see Table 1). For these values of the σ the relation $m_{Z'}/m_Z$ and $\sin\theta_m$ as functions of the $\Delta\rho_0 \equiv \rho_0 - 1$ are presented on Fig.1. Taking the current value $\rho_0 = 1.0004 \pm 0.0022 \pm 0.002$ [5] into account we see from Fig.1 that the Z'-boson may be rather light. Thus for $M_C = 10^6\, GeV$ and $\Delta\rho < 0.002$ we get the limits $m_{Z'} > 5\, m_Z$ on Z'-mass and $\theta_m < 0.01$ on the Z-Z' mixing angle. This upper limit on θ_m is compatible with those obtained in the extended gauge models [6, 7, 8].

Using the structure (5) - (9) of the neutral current interaction we have calculated in the tree approximation the cross sections $\sigma_{\bar{f}f} = \sigma(e^+e^- \to \gamma, Z, Z' \to \bar{f}f)$. The leptonic cross section $\sigma_{\bar{\ell}\ell}$ is found to be less than the one predicted by the SM. This effect is due to the destructive γ-Z'-interference [9]. The magnitude of this deviation depends on the MQLS-model parameters $m_{Z'}$ and M_C. For instance, at $M_C = 10^6\, GeV$ the relative deviation $\delta_{\bar{\ell}\ell} = (\sigma_{\bar{\ell}\ell} - \sigma_{\bar{\ell}\ell}^{SM})/\sigma_{\bar{\ell}\ell}^{SM}$ of the leptonic cross section $\sigma_{\bar{\ell}\ell}$ from the SM prediction $\sigma_{\bar{\ell}\ell}^{SM}$

at the TRISTAN energies ($\sqrt{s} \simeq 60\,GeV$) is about $\delta_{\bar{\ell}\ell} \simeq -6\%$ for $m_{Z'} = 4m_Z$ and $\delta_{\bar{\ell}\ell} \simeq -1\%$ for $m_{Z'} = 10m_Z$. These deviations are of the same order as the experimental errors of the leptonic cross section measurements at TRISTAN. The current values of the measured leptonic cross sections at TRISTAN are still slightly lower but they are consistent with the SM prediction [10]. The measurements of the leptonic cross sections with 2% accuracy which is to be achieved soon at TRISTAN can give additional limits on $m_{Z'}$ and M_C. At the LEP200 energies ($\sqrt{s} \simeq 200\,GeV$) these deviations are significantly larger and reach the values $\delta_{\bar{\ell}\ell} \simeq -60\%$ and $\delta_{\bar{\ell}\ell} \simeq -10\%$ for $m_{Z'} = 4m_Z$ and $10m_Z$ respectively. Hence the measurements of the leptonic cross sections $\sigma_{\bar{\ell}\ell}$ at LEP200 will allow either to observe the manifestation of the Z'-boson originated from the four-color quark-lepton symmetry or to obtain the more stringent limits on the MQLS-model parameters $m_{Z'}$ and M_C.

The hadronic cross section $\sigma_h = \sum_q \sigma_{\bar{q}q}$ is found to be somewhat more than that predicted by the SM but this deviation is smaller than that in the leptonic case. For instance, at $M_C = 10^6\,GeV$ the relative deviation $\delta_h = (\sigma_h - \sigma_h^{SM})/\sigma_h^{SM}$ at TRISTAN energies is only about $\delta_h \simeq 1.2\%$ for $m_{Z'} = 4m_Z$ and does not exceed the experimental errors at TRISTAN. At the LEP200 energies this deviation is about $\delta_h \simeq 20\%$ for $m_{Z'} = 4m_Z$ and $\delta_h \simeq 1.4\%$ for $m_{Z'} = 10m_Z$ and hence at 5% accuracy may be also observable if $m_{Z'} \sim 400 \div 600\,GeV$. As seen the measurements of the leptonic cross sections are more favourable for the search for the possible manifestations of the extra Z'-boson than the measurements of the hadronic cross sections.

In conclusion we resume the results of the work. The minimal quark-lepton symmetry model of the unification of the strong and electroweak interactions as a minimal extension of the Standard Model containing the four-color quark-lepton symmetry is proposed. The new relation between the masses of the W-, Z - and Z'-bosons is obtained. The existence of a rather light extra Z'-boson originated from the four-color quark-lepton symmetry is shown to be compatible with the current electroweak data.

Taking the structure of the neutral current interaction specified by the MQLS-model into account the cross sections of e^+e^- annihilation into leptons and hadrons are calculated and analysed. The deviations of these cross sections from the SM predictions are shown to be rather significant and available for the measurements at LEP200 and future colliders. These measurements will allow either to observe the manifestation of the Z'-boson originated from

the four-color quark-lepton symmetry or to obtain the more stringent limits on the mass of Z'-boson and on the mass scale of the four-color symmetry breaking.

Acknowledgments

The author is grateful to N.V. Mikheev, A.A. Gvozdev and L.A. Vassilevskaya for discussions of the results and to A.Ya. Parkhomenko for the help in the work. The work was supported by the Russian Foundation for Fundamental Research (Grant No. 93-02-14414).

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Table 1: The strong mixing angle $\sin^2\theta_S$ and the parameter σ depending on the mass scale M_C in the MQLS-model.

M_C, GeV	$\sin^2\theta_S$	σ
10^6	0.130	0.216
10^{10}	0.213	0.297
10^{14}	0.297	0.380

Figure caption

Fig. 1. Mass relation $m_{Z'}/m_Z$ and $\sin\theta_m$ as functions of the $\Delta\rho_0$ in MQLS-model: a) $M_C = 10^6\,GeV$, b) $M_C = 10^{10}\,GeV$, c) $M_C = 10^{14}\,GeV$.